Lebanese University
Faculty of information
Section 1 (2020/2021)

Final Exam: Discrete Mathematics **Time: 90 minutes**

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Problem 1.

1. Use truth tables to show that the following are tautologies.
   a) $[p \wedge (p \to q)] \to q$
   b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$
2. Show that the statements $(p \wedge \neg r) \to \neg q \equiv p \to (q \to r)$ are logically equivalent.
3. Show that the statements are not logically equivalent.
   a) $[p \to (q \to r)] \not\equiv [(p \to q) \to r]$
   b) $(p \to q) \not\equiv (\neg p \to \neg q)$
4. Let $f: \mathrm{E} \to \mathrm{F}$ application. Let A and B be two parts of E.
   a) Show that if f is injective : $f(\bar{A}) \subseteq \overline{f(A)}$
   b) Show that if f is surjective $\overline{f(A)} \subseteq f(\bar{A})$
5. Let f be the function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x) = x^4 - 1$. Find
   a) $f^{-1}(\{0\})$
   b) $f^{-1}(\{x/x > 15\})$
   c) $f^{-1}(\{x/0 < x < 63\})$
6. Suppose A are B are sets. Show that:

$$(\mathrm{B} - \mathrm{A}) \subseteq \bar{A} \cap B$$

Problem 2.

1. Suppose that a and b are two integers such that $a \equiv 11 \; mod \; 19$ and $b \equiv 3 \; mod \; 19$. Find the integer c with $0 \leq c \leq 18$ such that:
   a) $c \equiv 13a \; mod \; 19$
   b) $c \equiv (a - b) mod \; 19$
   c) $c \equiv 2a^2 + 3b^2 \; mod \; 19$
2. Show that if p is prime, the only solutions of $x^2 \equiv 1 \pmod{p}$ are integers x such that $x \equiv 1 \pmod{p}$ or $x \equiv -1 \pmod{p}$.
3.
   a) Find an inverse of a modulo m for each of these pairs of relatively prime integers.
   $$a = 15, m = 52.$$
   b) Solve each of these congruences using the modular inverses found in part (a)
   $$15x \equiv 31 \pmod{52}$$

$1 = 15 \cdot 1 - 2 \cdot 52 + 6 \cdot 15$

p | q | p → q

Problem 3.

1. Consider the congruence modulo m relation $R = \{(a,b) \in \mathbb{Z} / \ 2(a-b) \in \mathbb{Z}\}$ be the relation on the set of real numbers.
   a) Show that the relation R is an equivalence relation on the set of integers.
   b) Describe the equivalence classes [0] and [1/4].

2. Let R_1 and R_2 be the "congruent modulo 3" and "the congruent modulo 8" relations, respectively, on the set of integers. That is,

$$R_1 = \{(a,b)/a \equiv b \ (\text{mod } 3)\}$$
$$R_2 = \{(a,b)/a \equiv b \ (\text{mod } 8)\}$$

   Find
   a) $R_1 \cup R_2$
   b) $R_1 \cap R_2$
   c) $R_1 - R_2$
   d) $R_1 \oplus R_2$

3. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

   Find the matrix representing
   a) $R_1 \cup R_2$ and $R_1 \cap R_2$
   b) $R_1 \circ R_2$
   c) $R_1 \oplus R_2$
   d) $\overline{R_2}$ and R_1^{-1}

Good Luck

$2(a-b) = 0$

$a = b$

$2(a-b) = 1/4$

$a-b = 1/8$

$S \in 0$

$S \in \frac{\text{odd}}{4}$